3D Modelling (INFODDM) - Exam B

Period 4, 2015-2016

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Exam Rules

1. The test consists of 7 questions on 4 pages, found on both sides of the paper.
2. You can get 100 points in total.
3. If you answer a question correctly, you will get the designated number of points. If you answer a question incorrectly, you will get no points. If you leave a (sub-)question unanswered, you will get 2 points.
4. Answer only in the designated boxes following the questions. All other sheets are scratch paper, and will not be checked. First solve the problems on scratch paper, and copy your solutions to this sheet.
5. No external material is allowed. You are provided with an auxiliary reference sheet with formulae.
6. Answer briefly, but completely; none of the questions in the exam require heavy computation or many details to reach a complete answer.
7. Good luck!

The Exam

1. (10 points) The following lemniscate is defined as a closed cubic uniform B-spline on six control points $p_1, p_2, \ldots, p_6$, creating six knots that divide the curve into segments $a, b, \ldots, f$.

![Lemniscate diagram]

- If we move control point $p_6$, which segments of the curve will change?
  
  **Segments $a, d, e,$ and $f$.**

- Which control points have been repeated to close the curve?
  
  **Control points $p_1, p_2,$ and $p_3$.**

2. **(12 points)** List three advantages or disadvantages of using a \( k \)-D-tree over a BSP-tree for storing a collection of triangles in \( \mathbb{R}^3 \).

<table>
<thead>
<tr>
<th>Advantage/Disadvantage</th>
<th>Description</th>
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<tr>
<td>1. A ( k )-D-tree is axis-aligned, which makes it easier to compute and test with.</td>
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<td>2. A ( k )-D-tree is balanced, and thus gives performance guarantees on queries.</td>
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<td>3. A BSP-tree needs to cut fewer triangles, which makes it smaller.</td>
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3. **(10 points)** The Doo-Sabin scheme and the Catmull-Clark scheme are two iterative schemes for subdividing quad meshes. Draw the result of a single Doo-Sabin iteration in the left figure, and the result of a single Catmull-Clark iteration in the right figure. You do not need to calculate exact coordinates.

![Doo-Sabin iteration](image1.png)

![Catmull-Clark iteration](image2.png)

4. **(14 points)** Prove that the sum of vertex-based angle defects \( \theta_v, \ v \in V \) (or: discrete Gaussian Curvatures) is \( 2\pi \chi(M) \), for mesh \( M = (V, E, F) \). Tip: use Euler characteristic.

The proof appears in Lecture 10, slides 12–13

5. **(10 points)** Explain why it is not possible for a parametrization to be both equiareal and conformal in the general case. Explain for which special surfaces this is possible.

A conformal parametrization preserves angles and orientation, and its first fundamental form is \( sI \), for \( s \in \mathbb{R}^+ \). An equiareal parametrization has \( \det(I) = 1 \). They both happen at the same time iff the first fundamental form is an identity. This only happens when the surface is developable.
6. (16 points) For the following two curves \( P \) and \( Q \), and distance \( d \), the resulting free-space diagram is shown.

(a) What does the diagram tell us about the Fréchet distance between \( P \) and \( Q \)?

The Fréchet distance is larger than \( d \), because there is no \( x,y \)-monotone path from the bottom left to the top right in the free space.

(b) The free-space diagram has a disconnected component. Explain this.

It means there are a segment of \( P \) and a segment of \( Q \) that are close to each other (indicated in the figure), but if we place two points on those segments, we cannot move them along the curves to the start or end without having a distance larger than \( d \) along the way.

7. (28 points) Answer 4 of these 8 questions *concisely* but fully. No formulas needed. Only the first 4 answered will be graded. Simple yes-no answers will be disregarded.

(a) What is the advantage of mean-value coordinates over cotangent weights?

cotangent weights can be negative arbitrarily. Mean value coordinates are guaranteed to be positive inside convex polygons.

(b) What is the difference between a non-contractible loop and a handle?

A handle is a special case of non-contractible loop. The difference is that if you cut out a handle, it does not break the mesh. Boundary loops are non-contractible, but still break the mesh.

(c) How does the Lagrange property of barycentric coordinates (interpolating the cage while deforming) makes conformal deformation impossible?

Since the deformation must interpolate the cage, the inside can have arbitrary conformal distortion. The best example is taking a square cage and making it into a parallelogram, and then the entire inside is a parallelogram as well.
(d) Describe an advantage and a disadvantage of MLS reconstruction over RBF (radial basis function).

**Advantage:** Solving a sparse and local problem in each point.

**Disadvantage:** only seeing a local environment (can miss details), and the radius needs to be calibrated accordingly.

(e) Why must the gradient of an energy $\nabla E$ be parallel to the gradient of a constraint $\nabla C$ in order to get a valid local critical point?

A critical point is where all the directional derivatives are zero in valid directions. Directional derivatives are zero in directions that are orthogonal to the gradient (unless it is zero on its own, and then all directional derivatives are zero). Thus, when the gradients of the energy and the constraints are parallel, valid directions, orthogonal to $\nabla C$, collide with zero directional derivatives of $E$.

(f) Explain the problem of a positive semidefinite energy function vs. just positive definite.

A semidefinite energy might have a continuum of local minima, which means that a solution is not a unique minimum in its local environment. That might lead to numerical and algorithmic instability. PD guarantees a unique local minimum (also a global minimum, since it is positive).

(g) A 1-ring is embedded completely in the plane. What is the gradient of the area minimization at the central vertex?

It is zero. The area is already minimal since the one ring is flat.

(h) What is the problem in Laplacian mesh deformation, that As-rigid-as-possible tries to solve?

Laplacian mesh deformation encodes details up to translation. That means that if the mesh is rotated, even globally, there is a shearing artefact. ARAP tries to reproduce the local 1-rings up to a rigid transformation, which is more general and detail preserving.