Commonsense Reasoning and Argumentation 2014/2015

HC 4
Circumscription (2)

Henry Prakken

Department of Information and Computing Sciences
Utrecht University, The Netherlands

16 February, 2015
Minimal models: definition (1)

With one minimised predicate.

Definition 1a [Model preference.] Let $P$ be a minimised predicate and $M_1$ and $M_2$ two models. $M_1 \leq^P M_2$ iff

- $M_1$ and $M_2$ have the same domain; and
- $I_{M_1}(P) \subseteq I_{M_2}(P)$.

Definition 1b [Minimal entailment.] Let $T^P$ be a theory $T$ with minimised predicate $P$. The formula $\varphi$ is minimally entailed by $T^P$ iff $\varphi$ is true in all $\leq^P$-minimal models of $T$. 
Minimising several predicates

**Example:** minimise $Ab_1$ and $Ab_2$ in

1. $\forall x ((Bird(x) \land \neg Ab_1(x)) \supset Canfly(x))$
2. $\forall x (Penguin(x) \supset Ab_1(x))$
3. $\forall x ((Penguin(x) \land \neg Ab_2(x)) \supset \neg Canfly(x))$
4. $Bird(t)$
Minimal models: definition (2)

With several minimised predicates.

Definition 2a [Model preference.] Let $P$ be a set of minimised predicates and $M_1$ and $M_2$ two models. $M_1 \preceq^P M_2$ iff

- $M_1$ and $M_2$ have the same domain; and
- $I_{M_1}(P) \subseteq I_{M_2}(P)$ for all $P \in P$.

Definition 2c [Minimal entailment.] Let $T^P$ be a theory $T$ with minimised predicates $P$. The formula $\varphi$ is minimally entailed by $T^P$ iff $\varphi$ is true in all $\preceq^P$-minimal models of $T$. 
Definition 2b [Circumscriptive theories.] Let $T$ be a theory and $P$ a set of minimised predicates. Then $T^P$ is a circumscriptive theory.
Unique-name axioms

- every object has a name; and
- every object has a unique name.

(Expressible in the object language if the domain is finite).
Default contraposition

1. Heteros are usually married
2. Gays are usually not married
3. Everybody is either hetero or gay (but not both)
4. John is not married

Does it follow (defeasibly) that John is gay?
Default contraposition (formal)

1. $\forall x((\text{Hetero}(x) \land \neg \text{Ab}_1(x)) \supset \text{Married}(x))$
2. $\forall x((\text{Gay}(x) \land \neg \text{Ab}_2(x)) \supset \neg \text{Married}(x))$
3. $\forall x(\text{Hetero}(x) \equiv \neg \text{Gay}(x))$
4. $\neg \text{Married}(j)$

$\{1 \rightarrow 4\} \models \{\text{Ab}_1, \text{Ab}_2\} \text{ Gay}(j)$?
Default contraposition (formal 2)

1. \( \forall x((\text{Hetero}(x) \land \neg \text{Ab}_1(x)) \supset \text{Married}(x)) \)
2. \( \forall x((\text{Gay}(x) \land \neg \text{Ab}_2(x)) \supset \neg \text{Married}(x)) \)
3. \( \forall x(\text{Hetero}(x) \equiv \neg \text{Gay}(x)) \)
4. \( \neg \text{Married}(j) \)
5. \( \forall x(\neg \text{Ab}_3(x) \supset \text{Hetero}(x)) \)

\( \{1 - 5\} \models \{\text{Ab}_1, \text{Ab}_2, \text{Ab}_3\} \implies \text{Gay}(j)? \)
Example 1.4.9 (simplified)

1. $\forall x((\text{Bird}(x) \land \neg \text{Ab}_1(x)) \supset \text{Flies}(x))$
2. $\forall x((\text{Penguin}(x) \supset \text{Ab}_1(x))$
3. $\forall x((\text{Penguin}(x) \land \neg \text{Ab}_2(x)) \supset \neg \text{Flies}(x))$
4. $\text{Penguin}(t)$

$\{1-\} \models^{\{\text{Ab}_1, \text{Ab}_2\}} \neg \text{Flies}(t)$?
Example 1.4.9

1. $\forall x((\text{Bird}(x) \land \neg \text{Ab}_1(x)) \Rightarrow \text{Flies}(x))$
2. $\forall x((\text{Penguin}(x) \supset \text{Ab}_1(x)))$
3. $\forall x((\text{Penguin}(x) \land \neg \text{Ab}_2(x)) \Rightarrow \neg \text{Flies}(x))$
4. $\forall x((\text{ObsasP}(x) \land \neg \text{Ab}_3(x)) \Rightarrow \text{Penguin}(x))$
5. $\forall x((\text{Penguin}(x) \supset \text{Bird}(x)))$
6. ObsasP($t$)

$\{1-\} \models^{\{\text{Ab}_1, \text{Ab}_2, \text{Ab}_3\}} \neg \text{Flies}(t)$?