Universiteit Utrecht

[Faculty of **Science**
Information and Computing Sciences]

Talen en Compilers

2022 - 2023, period 2

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1. Introduction

1.1 Course Organization

Course website

Website, reachable from the Education page:

`www.cs.uu.nl/docs/vakken/b3tc/`

Communication: sending updates by e-mail via Blackboard

Assignments

Three practicals:

- P0: refresh your FP, doesn't count for the final grade
- P1–P3: theoretical and practical aspects
- Work in groups of two, self organize

See `www.cs.uu.nl/docs/vakken/b3tc`

Exams

Two exams: T1, T2

- Contents for each is specified in the schedule
- You cannot use lecture notes or other material for the exams.

Resit (aanvullende toets) exam: T3

- You will receive an e-mail that tells you if you qualify for resit/relab, and telling you what you should in fact do. See the Osiris website for the rules.

Course Content Overview

Languages . . .

A **language** is a set of "correct" sentences.

- But what does that mean?

Languages . . .

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- Are all languages equally difficult or complicated?
- How can one decide whether a sentence is correct?
- How can one represent a correct sentence?

. . . and compilers

A **compiler** translates one language into another (possibly the same). How?

- get hold of the structure of the input program

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- get hold of the structure of the input program
- attach semantics to a sequence of symbols
- check whether a program makes sense
- optimize
- generate good machine code

Languages, grammars, meaning, and life

Computer science studies information processing.

- We describe and transfer **information** by means of **language**

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- Information is obtained by assigning **meaning** to **sentences**
- The **meaning** of a sentence is inferred from its **structure**
- The **structure** of a sentence is described by means of a **grammar**
- Grammars and languages are essential for many human activities: verbal and written communication, musical scores, DNA, and also programming

In this course

- Classes ("difficulty levels") of languages

In this course

- Classes ("difficulty levels") of languages
  - context-free languages

In this course

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- How to go from syntax to semantics

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- To **analyse** grammars to see whether or not specific properties hold;
- To **compose** components such as parsers, analysers, and code generators;
- To **apply these techniques** in the construction of all kinds of programs;
- To **explain** and **prove** why certain problems can or cannot be described by means of formalisms such as context-free grammars or finite-state automata.

Haskell

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Formal languages	Haskell
alphabet	datatype
sequence	list type
sentence/word	a concrete list
abstract syntax	datatype
grammar	parser
grammar transformation	parser transformation
parse tree	value of abstract syntax type
semantics	fold function, algebra

1.3 Haskell Refresh

Functions

Pattern matching

$$
\begin{aligned}
&\mathsf{length} :: [\mathsf{a}] \to \mathsf{Int} \\
&\mathsf{length}\ [\,] \quad\ = 0 \\
&\mathsf{length}\ (\mathsf{x} : \mathsf{xs}) = 1 + \mathsf{length}\ \mathsf{xs}
\end{aligned}
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Functions

Pattern matching and recursion.

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length :: [a] → Int
length []       = 0
length (x : xs) = 1 + length xs
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Type signatures, but type inference.

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$$

Type signatures, but type inference.

Polymorphism – length works for any list.

Currying

Functions with multiple arguments are written as "functions to functions to functions . . .":

$$
\begin{aligned}
&(+\!\!+) :: [a] \to [a] \to [a] \\
&[] \quad\quad +\!\!+ ys = ys \\
&(x : xs) +\!\!+ ys = x : (xs +\!\!+ ys)
\end{aligned}
$$

Again, $(+\!\!+)$ is polymorphic. We need not know the type of list elements, but both argument lists must have the same type of elements!

Higher-order functions – map

Applying a function to every element of a list:

$$\text{map} :: (a \to b) \to [a] \to [b]$$

Higher-order functions – map

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$$\mathsf{map} :: (\mathsf{a} \to \mathsf{b}) \to [\mathsf{a}] \to [\mathsf{b}]$$

Example:

$$\begin{aligned} \mathsf{map}\ (+1)\ [1,2,3,4,5] \\ = \quad [2,3,4,5,6] \end{aligned}$$

Higher-order functions – filter

Filtering a list according to a predicate:

$$\text{filter} :: (a \to \text{Bool}) \to [a] \to [a]$$

Higher-order functions – filter

Filtering a list according to a predicate:

> filter :: (a → Bool) → [a] → [a]

Example:

> filter even $[1, 2, 3, 4, 5]$
> $= \quad [\ \ 2, \quad 4\ \ ]$

Higher-order functions – foldr

Traversing a list according to its structure:

```
foldr :: (a → b → b) → b → [a] → b
```

Higher-order functions – foldr

Traversing a list according to its structure:

$$\text{foldr} :: (a \to b \to b) \to b \to [a] \to b$$

Example:

$$\begin{aligned}
& \text{foldr}\ (+)\ 0\ [1, \quad 2, \quad 3, \quad 4, \quad 5\ ] \\
= {} & \text{foldr}\ (+)\ 0\ (1 : \ 2 : \ 3 : \ 4 : \ 5 : \ []) \\
= {} & \text{foldr}\ (+)\ 0\ (1 : (2 : (3 : (4 : (5 : [])))) \\
= {} & \quad 1 + (2 + (3 + (4 + (5 + 0)))) \\
= {} & \quad 15
\end{aligned}$$

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Datatypes

```
data Tree a = Leaf a
            | Node (Tree a) (Tree a)
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Multiple **constructors**:

```
Leaf :: a → Tree a
Node :: Tree a → Tree a → Tree a
```

Constructors describe the shape of values of the datatype. They can be used in patterns.

Functions on trees

```
size :: Tree a → Int
size (Leaf x)   = 1
size (Node l r) = size l + size r
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Exercise: A function that reverses (mirrors) a tree.

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Exercise: A function that reverses (mirrors) a tree.

```
reverse :: Tree a → Tree a
reverse (Leaf x)   = Leaf x
reverse (Node l r) = Node (reverse r) (reverse l)
```

1.4 (Formal) Languages

What is a language?

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Which sentences belong to a language, and why?

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Which sentences belong to a language, and why?

- In natural languages, this is often informally defined and subject to discussion.
- For a formal language, we want a precise definition.

Sets

A **set** is a collection of elements.

- No duplicates
- No order
- The empty set: $\emptyset$
- A nonempty set: $\{a, b, c\}$

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- Union
- Intersection

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An **alphabet** is a (finite) set of symbols that can be used to form sentences.

- $\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$

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- ε

The empty sequence is difficult to visualize. Therefore, we usually write ε as a placeholder to denote the empty sequence.

Sequences, inductively

Given an arbitrary sequence over elements of a set A, we can make one of the two following observations:

- it is the empty sequence ε,

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- the sequence has a first element $a \in A$, and if we split off that element, the tail is still a (possibly empty) sequence z.

We can use this observation to define sequences.

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Given a set A. The set of **sequences over** A, written A^*, is defined as follows:

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In such an inductive definition, it is implicitly understood that

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Given a set A. The set of **sequences over** A, written A^*, is defined as follows:

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- if $a \in A$ and $z \in A^*$, then az is in A^*.

In such an inductive definition, it is implicitly understood that

- nothing else is in A^*,
- we can only apply the construction steps a finite number of times, i.e., only finite sequences are in A^*.

Remarks about sequences

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- How many elements does $\emptyset$ contain?
- How many elements does $\emptyset^*$ contain?
- How many elements does $\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ contain?
- How many elements does $\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^*$ contain?

Language

Given an alphabet A, a **language** is a subset of A^*.

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Note that we consider any set X to be a subset of itself: $X \subseteq X$.

So A^* is a valid language with alphabet A.

How to define a language?

So a language is just the set of correct sentences.

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But how do we define such a set?

- By enumerating all elements?
- By using a predicate?
- By giving an inductive definition?
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All these are possible, and more.

Example

Let the set of digits $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be our alphabet.

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This is a language:

> $L = \{2, 3, 5, 7, 11, 13, 17, 19\}$

How can we describe this language?

Example

Let the set of digits $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be our alphabet.

This is a language:

> $L = \{2, 3, 5, 7, 11, 13, 17, 19\}$

How can we describe this language?

- The language L is the language over D of all prime numbers less than 20.

Languages by enumeration

- Enumerating all elements of a language is impossible if the language is infinite.

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- Most interesting languages are infinite:
  - C#
  - Haskell
  - ...

Languages by enumeration

- Enumerating all elements of a language is impossible if the language is infinite.
- Most interesting languages are infinite:
  - C#
  - Haskell
  - ...
- Defining a language using a predicate seems better.

Defining by predicate example

Let $A = \{a, b, c\}$ be our alphabet.

Defining by predicate example

Let $A = \{a, b, c\}$ be our alphabet.

Then

$$PAL = \{s \in A^* \mid s = s^R\}$$

is the language of **palindromes** over A.

Example – contd.

Palindromes can also be defined inductively:

- ε is in PAL,
- a, b, c are in PAL,
- if P is in PAL, then aPa, bPb and cPc are also in PAL.

By predicate vs. by induction

Which definition is better?

> $\mathsf{PAL} = \{\mathsf{s} \in \mathsf{A}^* \mid \mathsf{s} = \mathsf{s}^R\}$

or

The set PAL of palindromes over A is defined as follows:

- ε is in PAL,
- a, b, c are in PAL,
- if P is in PAL, then aPa, bPb and cPc are also in PAL.

By predicate vs. by induction

Definition by predicate is (in this case) shorter.

By predicate vs. by induction

Definition by predicate is (in this case) shorter.

How can we check whether a given sequence is in PAL?

By predicate vs. by induction

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How can we check whether a given sequence is in PAL?

How can we generate all the words in PAL?

By predicate vs. by induction

Definition by predicate is (in this case) shorter.

How can we check whether a given sequence is in PAL?

How can we generate all the words in PAL?

An inductive definition gives us more structure, and is self-contained, making it easier to explain **why** a sentence is in the language.

Summary

Alphabet A finite set of symbols.

This werkcollege: Haskell setup, P0.

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Language A set of words/sentences, i.e., sequences of symbols from the alphabet.

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Summary

Alphabet	A finite set of symbols.
Language	A set of words/sentences, i.e., sequences of symbols from the alphabet.
Grammar	Next lecture: A way to define a language inductively by means of rewrite rules.

This werkcollege: Haskell setup, P0.