The effect of rating-based matchmaking on convergence to true rating in the Elo system

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Abstract

In this paper, we examine the interaction between the Elo rating system and rating-based matchmaking. Computer simulations are used to study convergence to true rating for matchmaking ranging from completely random to matching people with their closest match. It was found that matching people based on their rating can cause extreme Elo rating divergence, which is not a desired property for such a system. The causes of this effect are analyzed, and it is shown how the magnitude of the effect changes, depending on parameters such as the matchmaking range and the number of players in the simulated system.

Introduction

The Elo rating system [1], invented by Arpad Elo, is one of the most widely used rating systems. Many other rating systems are based on this system [2]. The main purpose of the system is to approximate players' skill relative to each other, but it is also often used to determine which players should play against each other. A lot of research has been done to optimize this matchmaking for specific games and real world scenarios [3][4][5]. Fundamentals about how matchmaking in general affects the performance of the Elo rating system have, however, not been extensively explored. This research aims to better understand that relationship. This will offer more insight into the workings of the Elo rating system and will provide a basis from which to build and analyze more complex matchmaking systems. The focus for this research lies entirely on the convergence to true rating of the Elo ratings and other matchmaking properties such as player enjoyment are not considered.

Methodology

To better understand how matchmaking affects Elo rating convergence, a simulation will be used. This means that the aforementioned relation can be studied without taking any external factors into account.

The matchmaking system works by taking a random Elo rating within a certain range \( r \) from the player's Elo rating. This range is clamped to the minimum and maximum Elo ratings of all the players. The player with an Elo rating closest to that will then become the former player's opponent.

After a player has been matched, both players play against each other. Each player \( i \) has a true rating \( t_i \), and, based on that rating, a win chance \( p_i \) is calculated. This happens in the same way the Elo rating system calculates the expected win chance based on the Elo rating of both players:

\[
p_i = \frac{1}{1 + 10^{(t_j - t_i)/400}}
\]

where \( t_i \) and \( t_j \) are the true ratings of players \( i \) and \( j \) respectively, and \( p_i \) is the win chance of player \( i \).

Using that win chance, a random number generator determines which player wins. It does that by getting a random number between 0 and 1 and checking if that number is higher than the win chance. The winner gains and the loser loses Elo rating as defined by Arpad Elo [1].

To research how the average difference between the true rating and the Elo rating of the players changes, the number of players and the Elo rating range in the matchmaking system will both be varied. Random players will go through the matchmaking system and play against an opponent. If there are \( N \) players, this happens \( N^x \times N \) times, where \( x \) is a parameter. Every \( N \) times, the average absolute difference between every player is stored.

The true rating of all the players is uniformly spread between 1000 and 2000, and all of the Elo ratings of the players start at 1500. Unless otherwise defined, there are 100 players, the matchmaking range is 0, and the K-factor is 24.
Results

The matchmaking system, as described before, is a spectrum of sorts. On one end is perfect matchmaking, with 'perfect' meaning that a player always plays against the opponent with an Elo rating closest to their own. This corresponds to a range of 0. On the other end is completely random matchmaking, where every opponent is chosen at random from the total playerbase. This is approached by making the range sufficiently high as to include all of the other players. Moving along the spectrum corresponds to changing the matchmaking range.

Perfect matchmaking as described above results in Elo ratings that at first converge to the players' true rating, but then diverge as shown in figure 1.

![Perfect matchmaking](image)

**Figure 1.** Average absolute difference between Elo rating and true rating, plotted against the average amount of games played per player. This graph is the result of averaging 10 simulations with 100 players and a K-factor of 24. The statistical error was always between 0 and 2.

Notable is that the rate of divergence slows down as more games are played, but it always keeps diverging. The effect seems to be logarithmic in nature.

It was found that players with a high true rating get higher Elo ratings over time, while the inverse is true for players with a low true rating. This can even cause players to attain negative Elo ratings.

Since perfect matchmaking shows such extreme results, it is of interest to find how this effect changes along the matchmaking spectrum. A test was performed for different matchmaking ranges which resulted in figure 2.

![Matchmaking ranges](image)

**Figure 2.** Average absolute difference between Elo rating and true rating, plotted against the average amount of games played per player. A plot is shown for five simulations with different matchmaking ranges. The second graph shows the average absolute difference after 3000 games for all ranges between 0 and 300. The graphs are the result of averaging 10 and 4 simulations respectively with 100 players and a K-factor of 24. The statistical error for each range was always between 0 and 3 for the first graph and between 0 and 6 for the second graph.

For ranges between 0 and 200, the effect is still at play, but for ranges from 200 upward, it seems to disappear. Here, the average absolute difference clearly converges to the true ratings as the matchmaking approaches complete randomness.

Real-world playerbases are often much larger than the simulated playerbase. Because of this, simulations were performed with varying amounts of players to study how the effect changes. It was found that changing the number of players used in the simulation changes the magnitude of the effect. This is shown in figure 3.
Discussion

It may seem odd that an intuitively 'perfect' matchmaking system would lead to such strange results. At first, the Elo ratings converge to the players' true ratings, as one would expect, but once that happens, the ratings start to diverge. To understand why this happens, the effect has to be investigated at a very small scale. Imagine an Elo system with three players with true and Elo ratings of 1200, 1300 and 1700 respectively, as shown in figure 4.

The divergence effect

<table>
<thead>
<tr>
<th>Players</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1200</td>
<td>1300</td>
<td>1700</td>
<td></td>
</tr>
<tr>
<td>Player 2 is eventually overestimated</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1099</td>
<td>1401</td>
<td>1700</td>
<td></td>
</tr>
<tr>
<td>Player loses rating on average</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>1099</td>
<td>1351</td>
<td>1750</td>
<td></td>
</tr>
<tr>
<td>System reaches new equilibrium</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1175</td>
<td>1275</td>
<td>1750</td>
<td>Equilibrium</td>
</tr>
</tbody>
</table>

Figure 4: A sequence of four Elo system states that explain why the divergence effect occurs.

State 1 is the equilibrium state where everyone's Elo rating is equal to their true rating. Within this state, player 2 will always play against player 1 and, on average, the system will remain exactly the same.

However, because the system is non-deterministic, it will eventually end up in state 2 where player 2 is overestimated. In this state, player 2 will end up playing against player 3. Since player 2 is overestimated, they will on average lose rating to player 3, which is represented by going to state 3.

In state 3, player 2 will once again end up playing against player 1. The Elo rating system will try to reach an equilibrium between player 1 and 2, however, player 1 and 2 together now have less total Elo rating to divide between themselves. This means the system will end up in state 4.

State 4 is similar to state one, because it is also an equilibrium. From this state, the whole process will repeat itself, which causes player 3's Elo rating to keep increasing while the Elo ratings of player 1 and 2 will keep decreasing.

Because of this effect, any differences in true rating within a system will be magnified over time, which causes the overall divergence.

The chance of state 2 occurring will, however, go down over time, because the players' Elo ratings are further apart. This means that the effect will slow down as seen in figure 1.

Of course, this is an extremely simplified explanation of this effect. Randomness means the effect is only a trend and it will fluctuate in reality. Once more players and ranges are involved it becomes more complex still.

It was found that the effect does not occur when using completely random matchmaking. This makes sense, because the effect originates from choosing opponents based on rating. However, it is a lot more interesting to see what happens in between perfect and completely random as captured by the different ranges in figure 2.

This figure shows that the effect disappears around the 200 range mark for the parameters that were used. One can also observe that the magnitude of the effect gradually decreases with an increasing range. This makes sense intuitively, but to fully understand this relationship, additional research will be required.

The number of players in the system also influences the effect as can be seen in figure 4. The magnitude of the effect increases as the number of players in the system grows. The explanation for this is that state 2, as described above, will occur more frequently if there are more players. This is because the average distance between player ratings is smaller.
Conclusion

It was shown that matchmaking based directly on Elo ratings can lead to unexpected and unwanted side-effects. This means that anyone using any matchmaking system based on Elo ratings should consider this effect and the possible implications it might have for their particular matchmaking system. In any real world application, the effect may be overshadowed by all of the other factors at play, which is why future research into this effect in real world scenarios would be a good idea.

Future research

The effect of varying parameters such as the amount of players in the system and the matchmaking range has been shown here, but how these parameters interact with each other is still unknown. Further research could aim to develop a better understanding of how exactly these parameters influence the effect.

Another interesting path for future research is quantifying how fun a matchmaking system is. While completely random matchmaking may be optimal for convergence, one can imagine that it is not enjoyable for real players playing with such a system. One could, for example, model the average length of win or lose streaks and use this value as an indicator for how fun a matchmaking system is to use.

In reality, matchmaking systems are often much more complex, because players are not always available, because players improve, or because of various other reasons. The hypothesis is that the effect discussed here is fundamentally present in any matchmaking system based on ratings of individual players in some capacity, but further research is necessary to determine if that is true and with what magnitude.

References


Supplementary Material

Statistics

To ensure getting valid results, every simulation was performed 10 times. Except for the simulation used to obtain the second graph in figure 2 which was only performed 4 times due to time constraints. The graphs show the averages of these simulations. Variance, standard deviation and statistical error were calculated for every point on the graph using the following formulas:

\[ \sigma^2 = \frac{\sum_{i=1}^{N} x_i^2}{N} - \mu^2 \]

\[ \epsilon = \frac{\sigma}{\sqrt{N-1}} \]

Where \( N \) is the number of simulations, \( x_i \) is the value for one of these simulations, \( \mu \) is the average, \( \sigma \) is the standard deviation, \( \sigma^2 \) is the variance and \( \epsilon \) is the statistical error.

It was found that showing this in the graphs was unnecessary because the statistical error is so small compared to the values that the lines overlap. Instead, the subtext for each graph notes the maximum statistical error found among all of the data points.