Solutions to exercises

Apologies for poor handwriting.
I probably made several (minor) mistakes.

1) \[
\begin{align*}
B \land A & \quad \lambda \cdot E \\
A & \\
B & \quad \lambda \cdot I \\
\hline
A \land B
\end{align*}
\]

2) \[
\begin{align*}
Q \rightarrow R & \\
Q & \quad \Rightarrow \cdot E \\
R & \quad \Rightarrow \cdot I \\
(\neg Q \rightarrow R) & \rightarrow R
\end{align*}
\]
3) \[
\frac{\overline{A^2} \overline{B^3}}{\neg(A \land B)} \quad \frac{A \land B}{\neg-E}
\]
\[
\frac{\bot}{\neg B} \quad \frac{\neg B}{\neg-I} \quad \Rightarrow-I 2
\]
\[
\frac{A \rightarrow \neg B}{\Rightarrow-I 1}
\]
\[
\neg(A \land B) \rightarrow (A \rightarrow \neg B)
\]
5) \[
\frac{A \land B^3}{A} \land \frac{A \rightarrow C^2}{C} = 8
\]
\[
\frac{(B \rightarrow 7C)^3}{A \land B^1} \land \frac{B}{7C} = 1
\]
\[
\frac{1}{7-1} = 3
\]
\[
\neg (B \rightarrow 7C) = 3 \rightarrow 1
\]
\[
(A \rightarrow C) \rightarrow \neg (B \rightarrow 7C) = 3 \rightarrow 1
\]
\[
(A \land B) \rightarrow ((A \rightarrow C) \rightarrow \neg (B \rightarrow 7C))
\]
\[
\frac{B \rightarrow \neg C}{B} \quad \frac{A \land B}{\neg C} \quad \frac{A \rightarrow C}{\neg E} \quad \frac{A \land B}{\neg E}
\]

\[
\neg C \quad \frac{A \rightarrow C}{\neg E} \quad \frac{A \land B}{\neg E}
\]

\[
\frac{1}{\neg (B \rightarrow \neg C)} \quad \frac{\neg (B \rightarrow \neg C)}{\neg I 3}
\]

\[
\frac{(A \rightarrow C) \rightarrow \neg (B \rightarrow \neg C)}{\neg I 2} \quad \frac{(A \land B) \rightarrow ((A \rightarrow C) \rightarrow \neg (B \rightarrow \neg C))}{\neg I 1}
\]
7) A \equiv \text{Alon likes kangaroos} \\
B \equiv \text{Betty likes frogs} \\
C \equiv \text{Carl likes hamsters}

From the assumptions:

\cdot A \land (B \lor C)
\cdot B \Rightarrow \neg A
\cdot C \Rightarrow B

Prove: \bot
8)
\[
\begin{align*}
\overline{A \lor B} & \\
\quad & \overline{A^2} \quad \overline{B^2} \\
\quad & \overline{BVA} \quad \overline{BVA} \\
\quad & \overline{V - E^2} \\
\quad & \overline{BVA} \\
\quad & \overline{I_1} \\
\hline
A \lor B & \Rightarrow \overline{BVA}
\end{align*}
\]

9)
\[
\begin{align*}
\overline{A \lor B} & \\
\quad & \overline{A} \quad \overline{A^3} \\
\quad & \overline{B} \quad \overline{B^3} \\
\quad & \overline{A \lor B^2} \\
\quad & \overline{1} \\
\quad & \overline{1} \\
\quad & \overline{1} \\
\quad & \overline{7 - I_2} \\
\quad & \overline{7 (A \lor B)} \\
\quad & \overline{(\overline{A} \lor \overline{B})} \\
\quad & \overline{\neg (A \lor B)} \\
\quad & \overline{S_1}
\end{align*}
\]
12) 

\[ \frac{A \rightarrow B}{\neg B} \rightarrow \neg e \]
\[ \frac{A \rightarrow B}{A} \text{ r.e.} \]
\[ \frac{\neg B}{\neg e} \]
\[ \frac{\neg e}{\neg A} \]
\[ \frac{\neg A}{\neg I \; 1} \]

\[ \neg A \leftrightarrow \neg B \]

13) 

\[ (\neg v \; 6) \rightarrow R \]
\[ \frac{\neg v \; 6}{P'} \text{ v.-I} \]
\[ \frac{R}{R} \text{ v.-I} \]
\[ R \]
\[ P \rightarrow R \]
14) \[ \frac{A \lor B}{C^1} \quad \text{\small I-I} \]
\[ (A \lor B) \land C \]
\[ \Rightarrow \text{I 1} \]
\[ C \Rightarrow (A \lor B) \land C \]

15) \[ \frac{\lor^2 \quad W \quad W \rightarrow X \quad \neg E}{x \quad \text{\small I-I}} \]
\[ x \lor z \]
\[ \lor^1 \quad W \lor y \quad \neg E \]
\[ x \lor z \]
\[ \Rightarrow \text{I 1} \]
\[ w \lor y \rightarrow x \lor z \]
16)

\[ \frac{A \lor (B \land A)}{A^2} \quad \frac{B \land A^2}{A} \quad \frac{A^2}{A} \]

\[ \Rightarrow \Sigma 1 \]

\[ A \lor (B \land A) \rightarrow A \]