Cycles and Paths vs functions

- Cycle c:
  - Take f(v,w) = 1 if (v,w) is an arc on the cycle
  - Take f(v,w) = 0 otherwise
- Note: this is a special case of a circulation
- Path p from s to t:
  - Take f(v,w) = 1 if (v,w) is an arc on the path
  - Take f(v,w) = 0 otherwise
- Note: this is a special case of a flow from s to t with value 1

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A useful theorem on circulations

- **Theorem** Let *f* be a circulation. Then *f* is a nonnegative linear combination of cycles in G.
  - Proof. Find a cycle c, with minimum flow on c *r*, and use induction with $f - r * c$.
  - *Note*: each step we have at least one additional arc (v,w) with $f(v,w) = 0$
- If *f* is integer, `*integer scalared*' linear combination.

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Corollary on flows

- **Theorem** Let *f* be a flow from *s* to *t*. *f* can be written as a linear combination of cycles and paths from *s* to *t*.
  - Look at the circulation by adding an edge from *t* to *s* and giving it flow *value(f)*. Then apply the previous result.
- If *f* is an integer flow, then *f* can be written as a linear combination of cycles and paths from *s* to *t* with each factor an integer.