Exercises Algorithms and Networks – Set VIII - 2010

Deadline: Tuesday, April 26, 2011, 13.00 hours.

Hand in the exercises legibly written or printed on paper. You can also put your work in the mailbox of Hans Bodlaender, which is in the coffee room of the 5th floor of the BBL. You can also send the exercises by email in this special case. (Paper is preferred.) In the latter case, put AN8 in the header of your email.

Question Do you allow your teacher to publish the combination of your notes and student number on the website of the course? (Yes - No?)

1. A kernel for Convex String Recoloring

Consider the following problem.

Given a set of colors C, an integer n, and a function $f : \{1, \ldots, n\} \rightarrow C$, we say that f is *convex*, if for each color $c \in C$, $1 \leq i < j \leq n$, we have that if $f(i) = f(j) = c$, then for all k, $i < k < j$, $f(k) = c$. In other words, for each color, the numbers with that color are consecutively.

For instance, $f(1)$ = red, $f(2)$ = white, $f(3)$ =white, $f(4)$ = blue, $f(5)$ = white; is NOT convex: look at position 4 and the white color.

We look at the following problem:

> **Given:** set of colors c, integer n, function $f : \{1, \ldots, n\} \rightarrow C$, integer k.
> **Parameter:** k
> **Question:** Is it possible to make f convex by changing for at most k numbers the colors?

In the example above, there is a solution for $k = 1$: give 4 the color white.

Note that this problem was also studied in Exercise Set 7.

Now, we will build a relatively small kernel for the Convex String Recoloring problem.

A *block* is a maximal sequence of numbers that all have the same color. In (red, white, white, blue, blue, white, white, blue) we have 5 blocks: first a red block (of size 1), then a white block (of size 2), then a blue block (of size two), then a white block (of size 2), and then a blue block (of size 1).

A color is *bad*, if it has at least two blocks. Otherwise, it is good.

(d) Argue that if there is a bad color with at least $k+2$ blocks, then there is no convex recoloring with at most k recolored numbers.

(e) Argue that each recoloring can decrease the number of bad colors by at most two.

(f) Argue if there are more than $2k$ bad colors, then there is no convex recoloring with at most k recolored numbers.

(g) Argue that if a block has size at least $k+1$, then if there is a convex recoloring with at most k recolored numbers, there is one that does not recolor any number in this block.

(h) Give a safe rule that ensures that each block has size at most $k+1$.

(i) Suppose we have a number of consecutive blocks, that all have a good color. Suppose the total size of these blocks is at least $k+1$. Argue that if there is a convex recoloring with at most k recolored numbers, there is one that does not recolor any number in this block.

(j) Give a safe rule that ensures that each set of consecutive blocks of good color have total size at most $k+1$.

(k) Formulate a kernelisation algorithm for the Convex String Recoloring problem that fulfills the conditions of kernels and yields resulting equivalent instances of size at most $O(k^3)$.

(l) Proof the $O(k^3)$ bound for your algorithm.

(m) (Harder question) Give a quadratic kernel for the Convex String Recoloring problem.

2. Independent dominating set on trees

A set of vertices $W \subseteq V$ is an *independent dominating set* in a graph $G = (V, E)$, if

1. for all $v, w \in W$: $\{v, w\} \notin E$ (i.e., W is an independent set), **and**
2. for all $v \in V - W$: there exists a $w \in W$ with $\{v, w\} \in E$ (i.e., W is a dominating set).

Give a linear time algorithm that computes the minimum size of an independent dominating set of a given **tree**.

Hint: Use dynamic programming.

3. Treewidth of series parallel graphs

A special type of graphs are the series parallel graphs.

Series parallel graphs have two special vertices, their *terminals*. G is a series parallel graph with terminals s, t, if one of the following three cases holds:

- G is a single edge $\{s, t\}$.

- There are series parallel graphs G' with terminals s', t', and G'' with terminals s'', t''. We obtain G by taking the disjoint union of G' and G'' and then identifying t' and s''. (Series-composition.)
- There are series parallel graphs G' with terminals s', t', and G'' with terminals s'', t''. We obtain G by taking the disjoint union of G' and G'' and then identifying s' and s'', and identifying t' and t''. (Parallel-composition.)

Show that series parallel graphs have treewidth at most two.

Bonus question: Modulator to single edges

Consider the following parameterized problem:

> MODULATOR TO DEGREE 2-GRAPH
> **Given**: Graph $G = (V, E)$, integer $k \geq 0$
> **Question**: Is there a set of vertices $W \subseteq V$ such that the graph obtained by removing all vertices in W consists of only vertices of degree 0 and 1 and $|W| \leq k$?
> **Parameter:** k

I.e., when W is removed, the remainder of the graph are single vertices and edges.

(i) Give a polynomial time approximation algorithm for this problem which has a constant ratio.

(Hint: consider a vertex of degree at least two; and look at it and two of its neighbors.)

(ii) Give a kernel for this problem which has polynomial size.