Facility Location Problems
Network Algorithms: Ideas and Techniques

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Universiteit Utrecht

This lecture

We look at the facility location problem, and discuss approaches for special cases — pointing to some lectures from this course

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Facility location

- Facilities: fire stations, hospitals, shops, guards, depots, police stations, ...
- Locations: places where we can put a facility (*candidate facility locations*) and places where users/customers are (*client locations*)
- It is desirable that users are close to a facility
- OR problem: where to place facilities, such that ...?

Facility location variants

- Variants of the problem
- Different costs
- Maximum distance to a facility
- Average distance to a facility
- Minimizing total cost
- Minimizing number of facilities
- Facilities with capacities
- . . .

Approaches

- Special cases
- Local search based algorithms (e.g., simulated annealing)
- (I)LP-based algorithms
- Combinatorial algorithms
- ...

Maximum distance and minimum number of facilities

- Given: road network, bound B
- Question: how to place as few facilities as possible, such that each location is at distance $\leq B$ to a facility

How do we model this as a combinatorial problem?

Making the model

- With an ALL PAIRS SHORTEST PATHS algorithm, compute all distances from candidate facility locations to client locations
- For each candidate facility location x, take the set S_x of all client locations that are at distance at most B from x
- SET COVER problem: take a set of facility locations X, with $|X|$ as small as possible, such that $\bigcup_{x \in X} S_x$ is the set of all client locations
- SET COVER is NP-hard (the decision variant is NP-complete), which implies ...
- Polynomial time approximation algorithm with ratio $O(\log n)$

A more special case

- If all locations are candidate facility location and client location, we have the DOMINATING SET problem
- **Given:** Graph $G = (V, E)$
- **Question:** Find a set W, with $|W|$ as small as possible, such that for all $v \in V$: $v \in W$ or v has a neighbor in W
- Take an edge in G if the vertices are at distance at most B
- DOMINATING SET is NP-hard
- A trivial algorithm solves it in $O(2^n \cdot n)$ time
- Faster exact algorithm: $O(1.5086^n)$ time: more on this in the lecture on exact algorithms

An Approximation Result

- Each candidate facility location has a cost
- Each client location has a distance to each facility location
- Distances fulfill triangle inequality
- Cost of serving a client by a facility at a location is proportional to distance
- Minimize total cost
- Shmoys, Tardos, Aardal (1998): approximation algorithm with ratio 3.16
- Several similar results for variants

If the number of facilities is small

- Suppose the maximum number of facilities we can place is a small constant K
- A trivial algorithm solves the problem in $O(n^{K+1})$ time (enumerate all sets)
- An $O(f(K)n^c)$ for some function f and constant c is *unlikely* to exist: the problem is hard for the class $W[2]$
- More on this in the lecture on *fixed parameter complexity*

Capacities for facilities

- Each candidate facility location x has a capacity $c(x)$
- After we specified on which candidate facility location, we have a facility, we must specify for each client location a facility that *serves* this client
- Each facility on location x can serve at most $c(x)$ clients

Evaluating a set of candidate locations

- Suppose we have specified the set of candidate locations
- Evaluating how good these are can be done with generalized (weighted) matching
- E.g.: can this set of locations serve each client such that distances are at most B?
- Or: what is the best assignment of clients to locations such that average distance is minimized
- See the lecture on matching
- Gives polynomial time algorithm if number of facilities is bounded

Corridor observance

- Suppose we have a building. 'Facilities' are guards
- A guard is placed on an intersection of corridors
- The guard observes all incident corridors
- How many guards do we need to observe all corridors

Vertex Cover

- VERTEX COVER: Given is a graph $G = (V, E)$
- **Question:** Find a set $W \subseteq V$ such that $|W|$ is as small as possible, and for each $\{v, w\} \in E$: $v \in W$ or $w \in W$
- VERTEX COVER models corridor observance problem
- VERTEX COVER is NP-complete
- If number of guards is at most K: $O(2^K(n+m))$ time algorithm: branching

Vertex Cover on trees

- If G is a tree, then VERTEX COVER can be solved in linear time with dynamic programming
  - For each v, let T_v be the subtree with v as root
  - Compute for all v: the minimum vertex cover of T_v with $v \in W$ and the minimum vertex cover with $v \notin W$
- Generalizes to larger class of graphs
- Many buildings have *bounded treewidth*: see the lecture on this topic

- Several versions of the problem have different combinatorial models
- Different algorithmic techniques help for the different versions
- The road: problem description $\Rightarrow$ model $\Rightarrow$ algorithm