2 – Facilities (slides 1 – 4)

Facility Location Problems
Network Algorithms: Ideas and Techniques

Winter 2008

Universiteit Utrecht

This lecture

The problem
Special cases and models
Maximum distance
Dominating set
An Approximation Result
Few facilities
Capacities for facilities
Corridor observance
Trees
Conclusions

We look at the facility location problem, and discuss approaches for special cases — pointing to some lectures from this course

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Facility location

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- Facilities: fire stations, hospitals, shops, guards, depots, police stations, . . .
- Locations: places where we can put a facility (*candidate facility locations*) and places where users/customers are (*client locations*)
- It is desirable that users are close to a facility
- OR problem: where to place facilities, such that . . . ?

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Facility location variants

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- Variants of the problem
- Different costs
- Maximum distance to a facility
- Average distance to a facility
- Minimizing total cost
- Minimizing number of facilities
- Facilities with capacities
- . . .

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Approaches

- Special cases
- Local search based algorithms (e.g., simulated annealing)
- (I)LP-based algorithms
- Combinatorial algorithms
- ...

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Maximum distance and minimum number of facilities

- Given: road network, bound B
- Question: how to place as few facilities as possible, such that each location is at distance $\leq B$ to a facility

How do we model this as a combinatorial problem?

Making the model

- With an All Pairs Shortest Paths algorithm, compute all distances from candidate facility locations to client locations
- For each candidate facility location x, take the set S_x of all client locations that are at distance at most B from x
- Set Cover problem: take a set of facility locations X, with $|X|$ as small as possible, such that $\bigcup_{x \in X} S_x$ is the set of all client locations
- Set Cover is NP-hard (the decision variant is NP-complete), which implies ...
- Polynomial time approximation algorithm with ratio $O(\log n)$

A more special case

- If all locations are candidate facility location and client location, we have the Dominating Set problem
- **Given:** Graph $G = (V, E)$
- **Question:** Find a set W, with $|W|$ as small as possible, such that for all $v \in V$: $v \in W$ or v has a neighbor in W
- Take an edge in G if the vertices are at distance at most B
- Dominating Set is NP-hard
- A trivial algorithm solves it in $O(2^n \cdot n)$ time
- Faster exact algorithm: $O(1.5086^n)$ time: more on this in the lecture on exact algorithms

2 – Facilities (slides 9 – 12)

An Approximation Result

- Each candidate facility location has a cost
- Each client location has a distance to each facility location
- Distances fulfill triangle inequality
- Cost of serving a client by a facility at a location is proportional to distance
- Minimize total cost
- Shmoys, Tardos, Aardal (1998): approximation algorithm with ratio 3.16
- Several similar results for variants

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If the number of facilities is small

- Suppose the maximum number of facilities we can place is a small constant K
- A trivial algorithm solves the problem in $O(n^{K+1})$ time (enumerate all sets)
- An $O(f(K)n^c)$ for some function f and constant c is *unlikely* to exist: the problem is hard for the class $W[2]$
- More on this in the lecture on *fixed parameter complexity*

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Capacities for facilities

- Each candidate facility location x has a capacity $c(x)$
- After we specified on which candidate facility location, we have a facility, we must specify for each client location a facility that *serves* this client
- Each facility on location x can serve at most $c(x)$ clients

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Evaluating a set of candidate locations

- Suppose we have specified the set of candidate locations
- Evaluating how good these are can be done with generalized (weighted) matching
- E.g.: can this set of locations serve each client such that distances are at most B?
- Or: what is the best assignment of clients to locations such that average distance is minimized
- See the lecture on matching
- Gives polynomial time algorithm if number of facilities is bounded

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Corridor observance

- Suppose we have a building. ‘Facilities’ are guards
- A guard is placed on an intersection of corridors
- The guard observes all incident corridors
- How many guards do we need to observe all corridors

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Vertex Cover

- Vertex Cover: Given is a graph $G = (V, E)$
- **Question:** Find a set $W \subseteq V$ such that $|W|$ is as small as possible, and for each $\{v, w\} \in E$: $v \in W$ or $w \in W$
- Vertex Cover models corridor observance problem
- Vertex Cover is NP-complete
- If number of guards is at most K: $O(2^K(n+m))$ time algorithm: branching

Vertex Cover on trees

- If G is a tree, then Vertex Cover can be solved in linear time with dynamic programming
  - For each v, let T_v be the subtree with v as root
  - Compute for all v: the minimum vertex cover of T_v with $v \in W$ and the minimum vertex cover with $v \notin W$
- Generalizes to larger class of graphs
- Many buildings have *bounded treewidth*: see the lecture on this topic

- Several versions of the problem have different combinatorial models
- Different algorithmic techniques help for the different versions
- The road: problem description ⇒ model ⇒ algorithm